Exercises for Lecture Course on Modelling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2014

Solution 7 - Modelling from first principles

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Task 1

From Newton's second law we have the relationship between the net external torque and the angular acceleration:

$$J\dot{\omega} = \tau_s + \tau_g$$

with τ_s and τ_g stand for spring torque and torque due to the gravity respectively

$$mL^2\dot{\omega} = k\alpha + mgL\sin\varphi$$

We know that $\alpha = \psi - \varphi$, then our system will be:

$$\begin{cases} \dot{\varphi} = \omega \\ \dot{\omega} = \dfrac{k(\psi - \varphi)}{mL^2} + \dfrac{g}{L}\sin\varphi \end{cases}$$

Let us define $\dot{\omega} = \frac{\omega(k+1)-\omega(k)}{\Delta T}$ and $\dot{\varphi} = \frac{\varphi(k+1)-\varphi(k)}{\Delta T}$ with ΔT the step size, it will be possible to define our model in the form:

$$x_{next} = x_{current} + f(x_{current}) \cdot \Delta T$$

$$\begin{cases} \varphi(k+1) = \varphi(k) + \omega(k) \cdot \Delta T \\ \omega(k+1) = \omega(k) + (\dfrac{k(\psi - \varphi(k))}{mL^2} + \dfrac{g}{L}\sin\varphi(k)) \cdot \Delta T \end{cases}$$

and here is the matricial form:

$$\begin{bmatrix} \varphi(k+1) \\ \omega(k+1) \end{bmatrix} = \begin{bmatrix} \varphi(k) \\ \omega(k) \end{bmatrix} + \begin{bmatrix} \omega(k) \\ \frac{k(\psi-\varphi(k))}{mL^2} + \frac{g}{L}\sin\varphi(k) \end{bmatrix} \cdot \Delta T$$