Exercises for Lecture Course on Modelling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2014

Exercise 5: Linear Least Squares (advanced)

(to be returned on Nov 25, 2014, 8:15 in HS 26, or before in building 102, 1st floor, 'Anbau')

Prof. Dr. Moritz Diehl and Robin Verschueren

Please remember to provide a solution on paper (written or typed) including all the necessary graphs from MATLAB. The MATLAB code (.m-files) should be sent to `robin.verschueren@gmail.com` and `giovanni@ampyxpower.com`

Aim of this sheet is to perform fitting of a non-linear curve with linear least squares and to become acquainted with recursive least squares.

Exercise Tasks

Linear least squares can also be used to fit other functions than straight lines through random data. In this question, we will look at a data set coming from the acceleration of a race car out of a resting position.

1. **Modeling the car:** (2 points)

   Consider an electric vehicle with a DC motor encoded with pulse width modulation (PWM). We can write a one-dimension car model as follows:

   $$\dot{v}_X(t) = C_1 D(t) - C_2 - C_3 v_X(t),$$

   where v_X [m] denotes the velocity the car along the X-axis, $D(-)$ is the dutycycle of the DC motor, and C_2 and C_3 are zeroth and first order coefficients describing the friction of the car, respectively.

   (a) Find an explicit solution for this linear ODE.

   (b) From the previous solution, find an explicit solution for the position $p_X(t)$ of the center of gravity of the race car.

2. **Fitting non-linear curves:** (5 points)

   Next, we obtain measurements on the velocity and position of the car. Idea is to fit the data in `data5.txt` using weighted linear least squares. The data text file is formatted as follows:
   `|time|position|velocity|`.

   The constants $C_2 = 0.1\,\mathrm{m\,s^{-1}}$ and $C_3 = 0.6\,\mathrm{s^{-1}}$ are known, while we try to estimate the remaining parameter and initial conditions, i.e. the vector $[C_1, p_X(0), v_X(0)]^\top =: \theta \in \mathbb{R}^3$. We assume that the measurement errors are i.i.d. and zero mean, and have standard deviations of $\sigma_p = 0.1$ and $\sigma_v = 0.05$ for position and velocity measurements. The duty cycle D was constant throughout the experiment and set to $D = 0.4$. The times have no errors.

   Following the maximum likelihood approach, choose the right regressors and weightings in order to estimate the parameter value. At the end, give the correct value for C_1 and the one-sigma confidence interval.

3. **Recursive Least Squares** (3 points)

   Perform recursive least squares on the above problem and compare the results with the results from the previous question 2. For starting up the recursive estimation procedure, assume a bit of prior knowledge on $\theta \in \mathbb{R}^3$, namely that it is normally distributed according to $\mathcal{N}(0, 100 \cdot \mathbb{I})$.

This sheet gives in total 10 points