Exercises for Lecture Course on Modelling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2014

Exercise 7: Modeling from first principles

(to be returned on Dec 9, 2014, 8:15 in HS 26, or before in building 102, 1st floor, 'Anbau')

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Please remember to provide a solution on paper (written or typed) including all the necessary graphs from MATLAB. The MATLAB code (.m-files) should be sent to
`robin.verschueren@gmail.com` and `giovanni@ampyxpower.com`

The aim of this sheet is to learn how to build models from scratch and use them in nonlinear system identification.

Exercise Tasks

Regard a ball on a rod, attached with a torsion spring to an axle. On the end of the axle, there is a small handle (see also Fig. 1).

1. Come up with a model to describe the position (angle φ) and the velocity (angular velocity ω) of the ball with weight m [kg]. The weight of the rod with length L [m] can be neglected, and the angle $\varphi = 0\,\mathrm{rad}$ is vertically down (as shown on Fig. 1). The angle of the handle is denoted by ψ [rad], where $\psi = 0\,\mathrm{rad}$ is vertically up (as depicted). Between the rod and the axle is a linear torsion spring, governed by the equation $\tau = -k \cdot \alpha$, where α [rad] the applied angle and τ [Nm] is the developed torsion. Note that the spring is at rest in Fig. 1. (5 points)

2. Download `data7.txt` from `www.bit.do/MSI_ex`. Given is a certain trajectory of the ball in the following format: `|time|angle|angular velocity|`. The following are known to be true:

$$
\begin{aligned}
m &= 1\,\mathrm{kg} \\
L &= 3\,\mathrm{m} \\
\psi &= \pi/2\,\mathrm{rad} \\
g &= 9.81\,\mathrm{m/s^2}.
\end{aligned}
$$

   It is your job to estimate the initial angle $\varphi(0)$ and initial angular velocity $\omega(0)$, as well as the spring constant k (so $\theta = [\varphi(0), \omega(0), k]^\top$). Do this with nonlinear least squares (`lsqnonlin` in MATLAB). You will have to use similar `simstep` and `simloop` functions as in Exercise 6, however, now there is no (easy) closed-form analytic formula to go from one step to the next. To do this, use the explicit Euler method of numerical integration:

$$x_{\text{next}} = x_{\text{current}} + f(x_{\text{current}}) \cdot \Delta T,$$

   where $f(x)$ is the right hand side of the ODE $\dot{x} = f(x)$ and ΔT is the step size. You can put this in a function `euler_step`. Invoke this function from a function called `euler_loop`, which does integration on the interval $t \in [0, N * \Delta T]\,\mathrm{s}$.

   In doing the nonlinear least squares estimation, start with a reasonable initial guess θ_0. If you have a bad initial guess, your algorithm might not converge to the right solution! (5 points)

This sheet gives in total 10 points and 0 bonus points

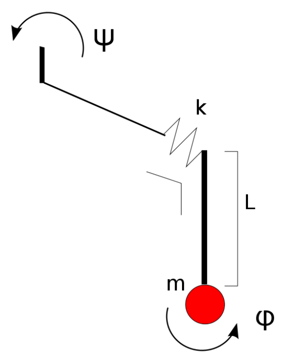

Abbildung 1: Sketch of pendulum.