

## Exercise 8 - Identification of a heating system: the Prediction Error Method

(to be returned on Dec 16, 2014, 8:15 in HS 26, or before in building 102, 1st floor, 'Anbau')

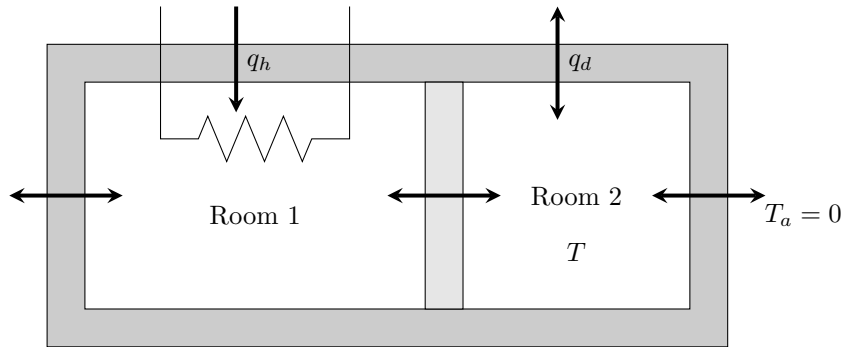
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Please remember to provide a solution on paper (written or typed) including all the necessary graphs from MATLAB. The MATLAB code (.m-files) should be sent to [robin.verschuereen@gmail.com](mailto:robin.verschuereen@gmail.com) and [giovanni@ampyxpower.com](mailto:giovanni@ampyxpower.com)

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The aim of this exercise is to identify the dynamics of the heating system inside the building depicted below:



The building is composed by two rooms and an electrical heater is installed only in Room 1 ( $q_h$  represents the power released by the heater to Room 1). In Room 2 there is an unknown flow  $q_d$  exchanging heat with the outside environment (e.g. sun shining into the windows). Both rooms exchange heat with each other and with the environment. We are interested in characterizing the temperature in Room 2, which we denote as  $T$ . We are here interested in a black-box model of ARX type. We make the following assumptions:

- the external ambient temperature is equal to zero during the experiments ( $T_a = 0$ );
- $q_d$  is considered as a white noise with zero mean.

An experiment is performed where the system is excited by some input values  $u(t) \equiv q_h(t)$  and where we measure the temperature  $y(t) \equiv T(t)$ . The equation errors are given by the random perturbations,  $\epsilon(t) \equiv q_d(t)$ .

### Exercise Tasks

1. Write a Matlab function which given an input vector  $\mathbf{u}$  and an output vector  $\mathbf{y}$  identifies the coefficient vectors  $\mathbf{a}$  and  $\mathbf{b}$  of the ARX model

$$y(t) + a_1 y(t-1) + a_2 y(t-2) + \dots + a_{n_a} y(t-n_a) = b_1 u(t-1) + b_2 u(t-2) + \dots + b_{n_b} u(t-n_b) + \epsilon(t)$$

using the prediction error method (PEM) explained in the lecture. The function should have the following syntax

```
[a, b] = arx_pem(y, u, na, nb);
```

(3 points)

2. Load inside Matlab the data contained in the file `ww.mat` using the command

```
load ww.mat
```

The file contains the following variables:

- $\mathbf{u}$  and  $\mathbf{y}$ : sampled input and output measured during the experiment;
- $\mathbf{t}$ : the vector containing the sampling times used in the experiment;

Check that the variables have been loaded correctly using the command `whos`.

3. Using the function created in Task 1 identify the model corresponding to data set. Set the order  $n_a = n_b = 2$  (the motivation for this is that we expect a second order model to describe the system well, because a state space model of the two rooms would have two states, the temperature in each room). Give the values  $a$  and  $b$ . (3 points)
4. With the ARX coefficients calculated in the previous task define the corresponding discrete-time transfer function (use the command `tf` and find the sampling time using the vector `t`). (2 points)
5. Use the identified system and the command `lsim` to simulate the output trajectory `ysim` resulting from the given input trajectory `u`. Compare `ysim` with the actual measurements `y` in a plot. (2 points)
6. \* Instead of minimizing the equation errors (giving rise to linear least squares) one could also minimize the output errors, which gives rise to nonlinear least squares. For this aim, write a function that simulates the linear system for given parameters  $a$ ,  $b$  and control trajectories (you can use `lsim`, as above), subtract the resulting output from the actual measurements, and minimize the sum of the squares of these residuals using `lsqnonlin`. For initialization, you can use the values obtained in Task 3. (3 bonus points)
7. Simulate and plot the response of the identified system model to the following input trajectory:

$$u(t) = \begin{cases} 0 & \text{if } t < 15h \text{ or } t > 80h \\ 200 & \text{else} \end{cases}$$

Note: Although `t` is indicated here in hours, the sampling time is defined in seconds. (2 points)

8. \* If you like, you can derive a first principles continuous time model of the building using the temperatures in room 1 and 2 as states, and assuming linear heat transfer between the rooms and with the environment. Give suitable names to the unknown constants. Formulate the model in state space form and then try to eliminate the temperature of room 1 in order to obtain a continuous time input-output model, which should be linear and time invariant. How many unknown parameters and which model order does the LTI input-output model have? (3 bonus points)

*This sheet gives in total 12 points and 6 bonus points*

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## Installation of Matlab via the Uni Freiburg Student Licence

Go to <https://www.rz.uni-freiburg.de/services/beschaffung/software/matlab-landeslizenz>. Go to the line where it says: **Bezug der Software (Student Option)**. There you find the link to the software shop. You have to register in that shop with your university e-mail. Afterwards you can download Matlab. Next, you have to make a MathWorks account - it is important that you do this with your university e-mail (something like [planet.uni-freiburg.de](mailto:planet.uni-freiburg.de)). Further information about the MathWorks registration you will get with your order in the software shop. You can install all available MATLAB toolboxes if your computer has enough memory. On the other hand, the toolboxes you really need for this course are:

- Control System Toolbox
- Optimization Toolbox
- System Identification Toolbox