

**Exercise 9: Simulation of a Heating System with Periodic Excitation**  
 (to be returned on Jan. 13., 2015, 8:15 in HS 26, or before in building 102, 1st floor, 'Anbau')

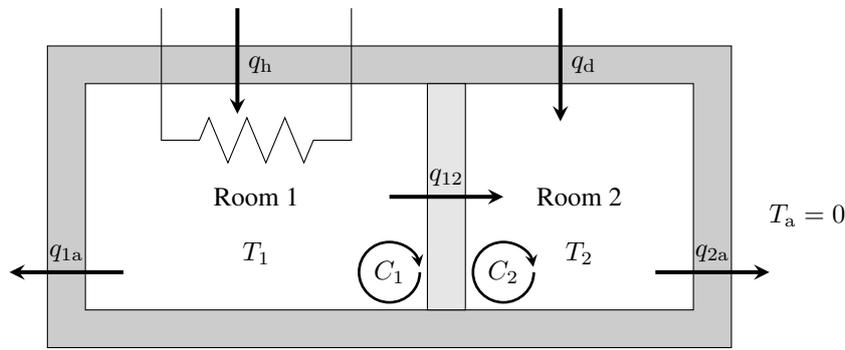
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Please remember to provide a solution on paper (written or typed) including all the necessary graphs from MATLAB. The MATLAB code (.m-files) should be sent to [robin.verschueren@gmail.com](mailto:robin.verschueren@gmail.com) and [giovanni@ampyxpower.com](mailto:giovanni@ampyxpower.com)

This exercise and Exercise Sheet 10 of next week belong together and are dedicated to the modelling and identification of a heating system. This week we model and simulate the system and next week we use our simulation model to simulate identification experiments that will allow us to estimate the frequency response function of the system.

**Introduction**

The aim of this exercise is to model and simulate the dynamics of the heating system inside the building depicted below:



The building is composed by two rooms and a heater is installed only in Room 1 ( $q_h$  represents the power released by the heater to Room 1). In Room 2 there is an unknown flow  $q_d$  exchanging heat with the external environment. We are interested in characterizing the temperature in Room 2, which we denote as  $T_2$ .

We make the following assumptions:

- the external ambient temperature is equal to zero during the experiments ( $T_a = 0$ );
- $q_d$  is modeled as a i.i.d. white Gaussian noise with zero mean.

The room temperature dynamics are characterized by the following equations

$$\begin{aligned} C_1 \frac{dT_1(t)}{dt} &= q_h(t) - q_{12}(t) - q_{1a}(t) \\ C_2 \frac{dT_2(t)}{dt} &= q_d(t) + q_{12}(t) - q_{2a}(t) \end{aligned} \tag{1}$$

$$q_{2a}(t) = \frac{T_2(t) - T_a(t)}{R_{2a}} \quad q_{1a}(t) = \frac{T_1(t) - T_a(t)}{R_{1a}} \tag{2}$$

$$q_{12}(t) = \frac{T_1(t) - T_2(t)}{R_{12}}$$

where

- $q_h(t)$  is the heat flow from the heater to Room 1;
- $q_{12}(t)$  is the heat flow from Room 1 to Room 2;
- $q_{1a}(t)$  is the heat flow from Room 1 to the environment;
- $q_d(t)$  is the disturbance heat flow from Room 2 to the environment;
- $q_{2a}(t)$  is the heat flow from Room 2 to the environment through the wall;
- $C_1$  is the thermal capacity of Room 1;
- $C_2$  is the thermal capacity of Room 2;
- $R_{12}$  is the thermal resistance between Room 1 and Room 2;
- $R_{1a}$  is the thermal resistance between Room 1 and the environment;
- $R_{2a}$  is the thermal resistance between Room 2 and the environment.

## Exercise Tasks

### Model the system

1. Write the system dynamics in the state-space form

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t)\end{aligned}$$

where

$$x(t) = [T_1(t) \ T_2(t)]^T, \quad u(t) = [q_h(t) \ q_d(t)]^T, \quad \text{and} \quad y(t) = T_2(t)$$

(here we treat  $q_d(t)$  as an input).

Hint: Use the equations (2) to substitute in (1) the heat flows which are not considered as inputs.

(2 points)

2. Construct the matrices A, B, C and D in Matlab using the following values for the constants

- $R_{1a} = 5 \times 10^{-3}$  [K/W];
- $R_{2a} = 10 \times 10^{-3}$  [K/W];
- $R_{12} = 10 \times 10^{-3}$  [K/W];
- $C_1 = 1 \times 10^6$  [J/K];
- $C_2 = 7 \times 10^5$  [J/K].

With the matrices created define the system `sys` using the command `ss`.

(1 point)

### Simulate the system

3. Generate an input signal to simulate an experiment. Set the sampling time `t_s` to 60 seconds. The experiment should last 6 days (therefore there should be 8640 samples). It consists of a periodic signal that is applied to the system three times (`n_periods=3`).

- $q_h(t)$  should be periodic with a period  $N=2880$  samples (which corresponds to 2 days). The signal should be the superposition of two sinusoidal curves with periods of 2880 samples and of 288 samples (the tenth harmonic). Scale the signal such that its value is contained in the interval  $[0, 6000]$ . Generate a plot of your input signal  $q_h(t)$  for the full time horizon. (2 points)
- $q_d(t)$  should be white Gaussian noise (use the function `wgn` and set the power to 25). Generate a plot of your disturbance signal  $q_d(t)$ . (1 point)

4. Simulate the system using the command `lsim` and plot the resulting output. (2 points)

5. Verify that in the last two days of the experiment the effects of the transient response are negligible. To do this you can consider the following signal

$$y\_diff = \text{abs}(y(1:(n\_periods-1)*N) - y(N+1:end))$$

where  $y$  is the output signal resulting from the simulation and `n_periods=3` is the number of repetitions of the input signal. If there was no disturbance, after the transient effects have disappeared, the output signal should be periodic of period  $N$ . Plot `y_diff` in logarithmic scale and verify that after  $N$  samples the transients have died out. (2 points)

*This sheet gives in total 10 points.*